Fair and Efficient Completion of Chores: An (Almost) Complete Picture

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Abstract

We study the completion setting recently introduced by HV et al. [2025], in which some items are already allocated and the goal is to allocate the remaining items so that the final allocation is fair and efficient. We consider indivisible chores in this work, and focus on additive cost functions as well as its subclasses such as binary and bi-valued costs. The fairness and efficiency notions of interest include MMS, PROP1, EF1 and PO. We present an (almost) complete picture of complexity and algorithmic results by investigating various combinations of the cost functions and notions.

1 Background

Fair allocation of indivisible items is a fundamental problem at the intersection of economics and computer science. Traditional fair allocation literature typically assumes that all items are unallocated prior to the allocation process. However, in many real-world scenarios, some agents already possess items or some items can only be allocated to specific agents. This mismatch may hinder the practical application of fair allocation outcomes.

To address the above challenge, HV *et al.* [2025] proposed the completion setting and studied indivisible goods, where every agent has a non-negative value for every good and wants to maximize the total value of the goods they receive. The problem is to determine that given a partial allocation of indivisible items, whether there exists an allocation of the remaining items (i.e., a completion) such that the aggregate allocation is both fair and efficient. In this work, we study the completion setting for indivisible chores, where every agent has a non-negative cost for every chore and wants to minimize the total cost of the chores they receive. This is highly motivated by the fact that many real-world scenarios—including the aforementioned examples—involve the completion of chores.

2 Our Contributions

We provide an (almost) complete picture of complexity and algorithmic results (as shown in Table 1). Our main contributions are as follows:

	Binary	**Bi-valued**	**Additive**
MMS	P	NP-c	NP-c
PROP1	P	NP-c	NP-c
EF1	NP-c	NP-c	NP-c
MMS+wPO	P	NP-c	NP-h
PROP1+wPO	P	NP-c	NP-h
EF1+wPO	NP-c	NP-c	NP-h
MMS+PO	P	NP-c	NP-h
PROP1+PO	P	?	?
EF1+PO	NP-c	NP-c	NP-h

Table 1: Summary of Our Results.

- For the share-based fairness notions MMS and PROP1, we identify a sharp complexity boundary between binary and bi-valued cost functions. For binary costs, we propose polynomial-time algorithms to determine the existence of MMS or PO completions, even when combined with PO or wPO. However, this tractability vanishes as soon as we move to bi-valued costs—a slightly broader class—where the problem becomes computationally intractable.
- For the envy-based notion EF1, we find that the completion problem (and when combined with PO or wPO) is NP-hard even under binary costs. This result underscores the challenge posed by the completion setting. Besides, it partially illustrates the difference between shared-based and envy-based notions.
- We find that the completion problem becomes highly intractable for general additive costs: it is NP-hard even for instances with only two agents or with identical cost functions. To complement these hardness results, we propose polynomial-time algorithms for certain special cases such as identical costs and for the approximation of the fairness notions.

References

Vishwa Prakash HV, Ayumi Igarashi, and Rohit Vaish. Fair and efficient completion of indivisible goods. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pages 14045–14053, 2025.